Minimum Mean-Squared Error Iterative Successive Parallel Arbitrated Decision Feedback Detectors for DS-CDMA Systems

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***Abstract*—n this paper we propose minimum mean squared error (MMSE) iterative successive parallel arbitrated decision feedback (DF) receivers for direct sequence code division multiple access (DS-CDMA) systems. We describe the MMSE design criterion for DF multiuser detectors along with successive, parallel and iterative interference cancellation structures. A novel efficient DF structure that employs successive cancellation with parallel arbitrated branches and a near-optimal low complexity user ordering algorithm are presented. The proposed DF receiver structure and the ordering algorithm are then combined with iterative cascaded DF stages for mitigating the deleterious effects of error propagation for convolutionally encoded systems with both Viterbi and turbo decoding as well as for uncoded schemes. We mathematically study the relations between the MMSE achieved by the analyzed DF structures, including the novel scheme, with imperfect and perfect feedback. Simulation results for an uplink scenario assess the new iterative DF detectors against linear receivers and evaluate the effects of error propagation of the new cancellation methods against existing ones.n this paper we propose minimum mean squared error (MMSE) iterative successive parallel arbitrated decision feedback (DF) receivers for direct sequence code division multiple access (DS-CDMA) systems. We describe the MMSE design criterion for DF multiuser detectors along with successive, parallel and iterative interference cancellation structures. A novel efficient DF structure that employs successive cancellation with parallel arbitrated branches and a near-optimal low complexity user ordering algorithm are presented. The proposed DF receiver structure and the ordering algorithm are then combined with iterative cascaded DF stages for mitigating the deleterious effects of error propagation for convolutionally encoded systems with both Viterbi and turbo decoding as well as for uncoded schemes. We mathematically study the relations between the MMSE achieved by the analyzed DF structures, including the novel scheme, with imperfect and perfect feedback. Simulation results for an uplink scenario assess the new iterative DF detectors against linear receivers and evaluate the effects of error propagation of the new cancellation methods against existing ones.I**

I. Introduction

Multiuser detection has been proposed as a means to suppress multi-access interference (MAI), increasing the capacity and the performance of CDMA systems [1]. The optimal multiuser detector of Verdu [2] suffers from exponential complexity and requires the knowledge of timing, amplitude and signature sequences. This fact has motivated the development of various sub-optimal strategies: the linear [3] and decision feedback (DF) [4] receivers, the successive interference canceller [5] and the multistage detector [6]. Recently, Verdu and Shamai [7] and Rapajic [8] *et al.* have investigated the information theoretic trade-off between the spectral and power efficiency of linear and non-linear multiuser detectors in synchronous AWGN channels. These works have shown that given a sufficient signal to noise ratio and for high loads (the ratio of users to processing gain close to one), DF detection has a substantially higher spectral efficiency than linear detection. For uplink scenarios, DF structures, which are relatively simple and perform linear interference suppression followed by interference cancellation, provide substantial gains over linear detection.

Minimum mean squared error (MMSE) multiuser detectors usually show good performance and have simple adaptive implementation. In particular, when used with short or repeated spreading sequences the MMSE design criterion leads to adaptive versions which only require a training sequence for estimating the receiver parameters. Previous work on DF detectors examined successive interference cancellation [9], [10], [11], parallel interference cancellation [13], [14], [15] and multistage or iterative DF detectors [14], [15]. The DF detector with successive interference cancellation (S-DF) is optimal, in the sense that it achieves the sum capacity of the the synchronous AWGN channel [10]. The S-DF scheme is capable of alleviating the effects of error propagation despite it generally leads to non uniform performance over the users. In particular, the user ordering plays an important role in the performance of S-DF detectors. Studies on decorrelator DF detectors with optimal user ordering have been reported in [11] for imperfect feedback and in [12] for perfect feedback. The problem with the optimal ordering algorithms in [11], [12] is that they represent a very high computational burden for practical receiver design. Conversely, the DF receiver with parallel interference cancellation (P-DF) [13], [14], [15] satisfies the uplink requirements, namely, cancellation of intra-cell interference and suppression of the remaining other-cell interference, and provides, in general, uniform performance over the user population even though it is more sensitive to error propagation. The multistage or iterative DF schemes presented in [14], [15] are based on the combination of S-DF and P-DF schemes in multiple stages in order to refine the symbol estimates, resulting in improved performance over conventional S-DF, P-DF and mitigation of error propagation.

In this work, we propose the design of MMSE DF detectors that employ a novel successive parallel arbitrated DF (SPA-

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DF) structure based on the generation of parallel arbitrated branches. The motivation for the novel DF structures is to mitigate the effects of error propagation often found in P-DF structures [13], [14], [15]. The basic idea is to improve the S-DF structure using different orders of cancellation and then select the most likely estimate. A near-optimal user ordering algorithm is described for the new SPA-DF detector structure and is compared to the optimal user ordering algorithm, which requires the evaluation of $K!$ different cancellation orders. The results in terms of performance show that the SPA-DF structure with the suboptimal ordering algorithm can achieve a performance very close to that of the S-DF with optimal ordering. Furthermore, the new SPA-DF scheme is combined with iterative cascaded DF stages, where the subsequent stage uses S-DF, P-DF or the new SPA-DF system to refine the symbol estimates of the users and combat the effects of error propagation. The performance of the proposed SPA-DF scheme and the sub-optimal ordering algorithm and their combinations with other schemes in a multistage detection structure is investigated for both uncoded and convolutionally encoded systems with Viterbi and turbo decoding.

This paper is structured as follows. Section II briefly describes the DS-CDMA system model. The MMSE decision feedback receiver filters are described in Section III. Sections IV is devoted to the novel SPA-DF scheme, the near-optimal user ordering algorithm and the combination of the SPA-DF detector with iterative cascaded DF stages and Section V details the proposed SPA-DF receiver for convolutionally coded systems with Viterbi and turbo decoding. Section VI presents and discusses the simulation results and Section VII draws the concluding remarks of this paper.

II. DS-CDMA SYSTEM MODEL

Let us consider the uplink of a symbol synchronous binary phase-shift keying (BPSK) DS-CDMA system with K users, N chips per symbol and L_p propagation paths. It should be remarked that a synchronous model is assumed for simplicity, although it captures most of the features of more realistic asynchronous models with small to moderate delay spreads. The baseband signal transmitted by the k-th active user to the base station is given by

$$x_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k(i) s_k(t - iT) \tag{1}$$

where $b_k(i) \in \{\pm 1\}$ denotes the i-th symbol for user k, the real valued spreading waveform and the amplitude associated with user k are $s_k(t)$ and A_k, respectively. The spreading waveforms are expressed by $s_k(t) = \sum_{i=1}^{N} a_k(i)\phi(t - iT_c)$, where $a_k(i) \in \{\pm 1/\sqrt{N}\}$, $\phi(t)$ is the chip waveform, T_c is the chip duration and $N = T/T_c$ is the processing gain. Assuming that the receiver is synchronised with the main path, the coherently demodulated composite received signal is

$$r(t) = \sum_{k=1}^{K} \sum_{l=0}^{L_p-1} h_{k,l}(t) x_k(t - \tau_{k,l}) + n(t) \tag{2}$$

where $h_{k,l}(t)$ and $\tau_{k,l}$ are, respectively, the channel coefficient and the delay associated with the l-th path and the k-th user. Assuming that $\tau_{k,l} = lT_c$, the channel is constant during each symbol interval, the spreading codes are repeated from symbol to symbol and the receiver is synchronized with the main path, the received signal $r(t)$ after filtering by a chip-pulse matched filter and sampled at chip rate yields the M-dimensional received vector

$$\begin{aligned}\mathbf{r}(i) &= \sum_{k=1}^{K} A_k b_k(i) \mathbf{C}_k \mathbf{h}_k(i) + A_k b_k(i-1) \bar{\mathbf{C}}_k \mathbf{h}_k(i-1) \\ &\quad + A_k b_k(i+1) \breve{\mathbf{C}}_k \mathbf{h}_k(i+1) + \mathbf{n}(i) \\ &= \sum_{k=1}^{K} \Big(A_k b_k(i) \mathbf{p}_k(i) + \boldsymbol{\eta}_k(i) \Big) + \mathbf{n}(i)\end{aligned} \tag{3}$$

where $M = N + L_p - 1$, $\mathbf{n}(i) = [n_1(i) \ \ldots \ n_M(i)]^T$ is the complex gaussian noise vector with $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma^2 \mathbf{I}$, $(.)^T$ and $(.)^H$ denote transpose and Hermitian transpose, respectively, $E[.]$ stands for ensemble average, $b_k(i) \in \{\pm 1 + j0\}$ is the symbol for user k, the amplitude of user k is A_k, the user k channel vector is $\mathbf{h}_k(i) = [h_{k,0}(i) \ldots h_{k,L_p-1}(i)]^T$ with $h_{k,l}(i) = h_{k,l}(iT_c)$ for $l = 0, \ldots, L_p - 1$, the ISI is given by $\boldsymbol{\eta}_k(i) = A_k b_k(i-1)\bar{\mathbf{C}}_k \mathbf{h}_k(i-1) + A_k b_k(i+1)\breve{\mathbf{C}}_k \mathbf{h}_k(i+1)$ and assumes that the channel order is not greater than N, i.e. $L_p - 1 \leq N$, $\mathbf{s}_k = [a_k(1) \ldots a_k(N)]^T$ is the signature sequence for user k and $\mathbf{p}_k(i) = \mathbf{C}_k \mathbf{h}_k(i)$ is the effective signature sequence for user k, the $M \times L_p$ convolution matrix $\mathbf{C}_k$ contains one-chip shifted versions of $\mathbf{s}_k$ and the $M \times L_p$ matrices $\bar{\mathbf{C}}_k$ and $\breve{\mathbf{C}}_k$ with segments of $\mathbf{s}_k$ have the following structure

$$\mathbf{C}_k = \begin{bmatrix} a_k(1) & 0 & \ldots & 0 \\ \vdots & a_k(1) & \ddots & \vdots \\ a_k(N) & \vdots & \ddots & 0 \\ 0 & a_k(N) & \ddots & a_k(1) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ddots & a_k(N) \end{bmatrix},$$

$$\bar{\mathbf{C}}_k = \begin{bmatrix} 0 & a_k(N) & \ldots & a_k(N - L_p + 1) \\ \vdots & 0 & \ddots & \vdots \\ 0 & \vdots & \ddots & a_k(N) \\ \vdots & 0 & \ddots & 0 \\ 0 & \vdots & \ddots & 0 \\ 0 & 0 & \ldots & 0 \end{bmatrix},$$

$$\breve{\mathbf{C}}_k = \begin{bmatrix} 0 & \ldots & 0 & 0 \\ \vdots & \ldots & \vdots & \vdots \\ 0 & \ldots & 0 & 0 \\ a_k(1) & \ddots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ a_k(L_p - 1) & \ldots & a_k(1) & 0 \end{bmatrix}.$$

The MAI comes from the non-orthogonality between the received signature sequences, whereas the ISI span L_s depends

on the length of the channel response, which is related to the length of the chip sequence. For $L_p = 1$, $L_s = 1$ (no ISI), for $1 < L_p \leq N, L_s = 2$, for $N < L_p \leq 2N, L_s = 3$.

III. MMSE Decision Feedback Receivers

Let us describe in this section the design of synchronous MMSE decision feedback detectors. The input to the hard decision device corresponding to the ith symbol is $\mathbf{z}(i) = \mathbf{W}^H(i)\mathbf{r}(i) - \mathbf{F}^H(i)\hat{\mathbf{b}}(i)$, where the input $\mathbf{z}(i) = [z_1(i) \ \ldots \ z_K(i)]^T$, $\mathbf{W}(i) = [\mathbf{w}_1 \ \ldots \ \mathbf{w}_K]$ is $M \times K$ the feedforward matrix, $\hat{\mathbf{b}}(i) = [b_1(i) \ \ldots \ b_K(i)]^T$ is the $K \times 1$ vector of estimated symbols, which are fed back through the $K \times K$ feedback matrix $\mathbf{F}(i) = [\mathbf{f}_1(i) \ \ldots \ \mathbf{f}_K(i)]$. Generally, the DF receiver design is equivalent to determining for user k a feedforward filter $\mathbf{w}_k(i)$ with M elements and a feedback one $\mathbf{f}_k(i)$ with K elements that provide an estimate of the desired symbol:

$$z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i) - \mathbf{f}_k^H(i)\hat{\mathbf{b}}(i) \ , \qquad k = 1, 2, \ldots, K \quad (4)$$

where $\hat{\mathbf{b}}(i) = \mathrm{sgn}[\Re(\mathbf{W}^H\mathbf{r}(i))]$ is the vector with initial decisions provided by the linear section, $\mathbf{w}_k$ and $\mathbf{f}_k$ are optimized by the MMSE criterion. In particular, the feedback filter $\mathbf{f}_k(i)$ of user k has a number of non-zero coefficients corresponding to the available number of feedback connections for each type of cancellation structure. The final detected symbol is:

$$\hat{b}_k^f(i) = \mathrm{sgn}\Big(\Re\Big[z_k(i)\Big]\Big) = \mathrm{sgn}\Big(\Re\Big[\mathbf{w}_k^H(i)\mathbf{r}(i) - \mathbf{f}_k^H(i)\hat{\mathbf{b}}(i)\Big]\Big) \quad (5)$$

where the operator $(.)^H$ denotes Hermitian transpose, $\Re(.)$ selects the real part and $\mathrm{sgn}(.)$ is the signum function.

To describe the optimal MMSE filters we will initially assume perfect feedback, that is $\hat{\mathbf{b}} = \mathbf{b}$, and then will consider a more general framework. Consider the following cost function:

$$J_{MSE} = E\Big[|b_k(i) - \mathbf{w}_k^H\mathbf{r}(i) + \mathbf{f}_k^H\mathbf{b}(i)|^2\Big] \quad (6)$$

Let us divide the users into two sets, similarly to [14]

$$D = \{j : \hat{b}_j \ is \ fed \ back \ \} \quad (7)$$

$$U = \{j : j \notin D\} \quad (8)$$

where the two sets D and U correspond to detected and undetected users, respectively. Let us also define the matrices of effective spreading sequences $\mathbf{P} = [\mathbf{p}_1 \ \ldots \ \mathbf{p}_K]$, $\mathbf{P}_D = [\mathbf{p}_1 \ \ldots \ \mathbf{p}_D]$ and $\mathbf{P}_U = [\mathbf{p}_1 \ \ldots \ \mathbf{p}_U]$. The minimization of the cost function in (6) with respect to the filters $\mathbf{w}_k$ and $\mathbf{f}_k$ yields:

$$\mathbf{w}_k = \mathbf{R}_U^{-1}\mathbf{p}_k \quad (9)$$

$$\mathbf{f}_k = \mathbf{P}_D^H\mathbf{w}_k \quad (10)$$

where the associated covariance matrices are $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)] = \mathbf{P}\mathbf{P}^H + \sigma^2\mathbf{I}$, $\mathbf{R}_U = \mathbf{P}_U\mathbf{P}_U^H + \sigma^2\mathbf{I} = \mathbf{R} - \mathbf{P}_D\mathbf{P}_D^H$. Thus, assuming perfect feedback and that user k is the desired one, the associated MMSE for the DF receiver is given by:

$$J_{MMSE} = \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}_U^{-1}\mathbf{p}_k \quad (11)$$

where $\sigma_b^2 = E[|b_k^2(i)|]$. The result in (11) means that in the absence of error propagation, the MAI in set D is eliminated and user k is only affected by interferers in set U.

For the successive interference cancellation DF (S-DF) detector , we have for user k

$$D = \{1, \ \ldots \ , k-1\}, \qquad U = \{k, \ \ldots \ , K\} \quad (12)$$

where the filter matrix $\mathbf{F}(i)$ is strictly upper triangular. The S-DF structure is optimal in the sense of that it achieves the sum capacity of the synchronous CDMA channel with AWGN [10]. In addition, the S-DF scheme is less affected by error propagation although it generally does not provide uniform performance over the user population. In order to design the S-DF receivers and satisfy the constraints of the S-DF structure, the designer must obtain the vector with initial decisions $\hat{\mathbf{b}}(i) = \mathrm{sgn}[\Re(\mathbf{W}^H(i)\mathbf{r}(i))]$ and then resort to the following cancellation approach. The non-zero part of the filter $\mathbf{f}_k$ corresponds to the number of used feedback connections and to the users to be cancelled. For the S-DF, the number of feedback elements and their associated number of non-zero filter coefficients in $\mathbf{f}_k$ (where k goes from the second detected user to the last one) range from 1 to $K-1$.

The parallel interference cancellation DF (P-DF) [14] receiver can offer uniform performance over the users but it suffers from error propagation. For the P-DF in a single cell, we have [14]

$$D = \{1, \ \ldots \ , k-1 \, k+1, \ \ldots, K\}, \qquad U = \{k\} \quad (13)$$

$$\mathbf{w}_k = \mathbf{R}_U^{-1}\mathbf{p}_k = \frac{\mathbf{p}_k}{A_k^2 + \sigma^2} \quad (14)$$

The MMSE associated with the P-DF system is obtained by substituting $\mathbf{R}_U = \mathbf{R} - \mathbf{P}_D\mathbf{P}_D^H$ into (9), which yields:

$$J_{MMSE} = \sigma_b^2 - \mathbf{p}_k^H(\mathbf{p}_k\mathbf{p}_k^H + \sigma^2\mathbf{I})^{-1}\mathbf{p}_k = \frac{\sigma^2}{A_k^2 + \sigma^2} \quad (15)$$

where for P-DF $\mathbf{F}(i)$ is full and constrained to have zeros along the diagonal to avoid cancelling the desired symbols. In order to design P-DF receivers and satisfy their constraints, the designer must obtain the vector with initial decisions $\hat{\mathbf{b}}(i) = \mathrm{sgn}[\Re(\mathbf{W}^H(i)\mathbf{r}(i))]$ and then resort to the following cancellation approach. The non-zero part of the filter $\mathbf{f}_k$ corresponds to the number of used feedback connections and to the users to be cancelled. For the P-DF, the feedback connections used and their associated number of non-zero filter coefficients in $\mathbf{f}_k$ are equal to $K-1$ for all users and the matrix $\mathbf{F}(i)$ has zeros on the main diagonal to avoid cancelling the desired symbols.

Now let us consider a more general framework, where the feedback is not perfect. The minimization of the cost function in (4) with respect to $\mathbf{w}_k$ and $\mathbf{f}_k$ leads to the following filter expressions:

$$\mathbf{w}_k = \mathbf{R}^{-1}(\mathbf{p}_k + \mathbf{B}\mathbf{f}_k) \quad (16)$$

$$\mathbf{f}_k = (E[\hat{\mathbf{b}}\hat{\mathbf{b}}^H])^{-1}\mathbf{B}^H\mathbf{w}_k \approx \mathbf{B}^H\mathbf{w}_k \quad (17)$$

where $E[\hat{\mathbf{b}}\hat{\mathbf{b}}^H] \approx \mathbf{I}$ for small error rates and $\mathbf{B} = E[\mathbf{r}(i)\hat{\mathbf{b}}^H(i)]$. The associated MMSE for DF receivers subject

to $E[\hat{\mathbf{b}}\hat{\mathbf{b}}^H] \approx \mathbf{I}$ and imperfect feedback is approximately given by

$$J_{MMSE} \approx \sigma_b^2 - \mathbf{p}_k^H \mathbf{R}^{-1} \mathbf{p}_k - \mathbf{p}_k^H \mathbf{R}^{-1} \mathbf{B} \mathbf{f}_k \qquad (18)$$

In Appendix I we show that the expression in (18) equals (11) under perfect feedback, and provide several other relationships between DF structure with and without perfect feedback. Note that the MMSE associated with DF receivers that are subject to imperfect feedback depends on the matrix $\mathbf{B} = E[\mathbf{r}\hat{\mathbf{b}}^H]$, that under perfect feedback equals $\mathbf{P}_D$, and the feedback filter $\mathbf{f}_k$ or set of filters $\mathbf{F}$. Specifically, if we choose a given structure for $\mathbf{F}$ this approach will lead to different methods of interference cancellation and performance improvements for the DF detector as compared to linear detection. The motivation for our work is to investigate alternative methods of finding structures for $\mathbf{F}$ that provide enhanced performance.

IV. Successive Parallel Arbitrated DF and Iterative Detection

In this section, we present a novel interference cancellation structure and describe a low complexity near-optimal ordering algorithm that employs different orders of cancellation and then selects the most likely symbol estimate. The proposed ordering algorithm is compared with the optimal user ordering algorithm, which requires the evaluation of $K!$ different cancellation orders and turns out to be too complex for practical use. The new receiver structure, denoted successive parallel arbitrated DF (SPA-DF) detection, is then combined with iterative cascaded DF stages [14], [15] to further refine the symbol estimates. The motivation for the novel DF structures is to mitigate the effects of error propagation often found in P-DF structures [14], [15], that are of great interest for uplink scenarios due to its capability of providing uniform performance over the users.

A. Successive Parallel Arbitrated DF Detection

The idea of parallel arbitration is to employ successive interference cancellation (SIC) to rapidly converge to a local maximum of the likelihood function and, by running parallel branches of SIC with different orders of cancellation, one can arrive at sufficiently different local maxima [16]. The goal of the new scheme, whose block diagram is shown in Fig. 1, is to improve performance using parallel searches and to select the most likely symbol estimate. The idea of the ordering algorithm is to employ SIC for different branches based on the power of the users to rapidly converge to a local maximum of the likelihood function and, on the basis of the euclidean distance, our approach selects the most likely estimate. In order to obtain the benefits of parallel search, the candidates should be arbitrated, yielding different estimates of a symbol. The estimate of a symbol that has the highest likelihood is then selected at the output. Unlike the work of Barriac and Madhow [16] that employed matched filters as the starting point, we adopt MMSE DF receivers as the initial condition and the euclidean distance for selecting the most likely symbol. The concept of parallel arbitration is thus incorporated into a DF detector structure, that applies linear interference suppression followed by SIC and yields improved starting points as compared to matched filters. Note that our approach does not require signal reconstruction as the PASIC in [16] because the MMSE filters automatically compute the coefficients for interference cancellation.

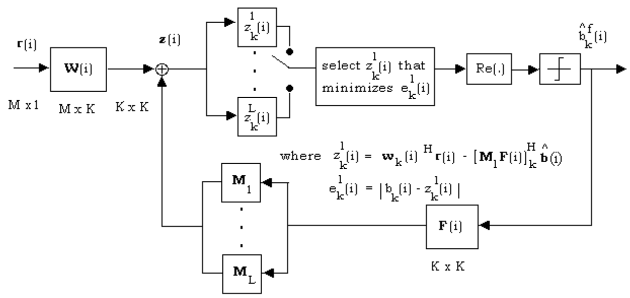

Fig. 1. Block diagram of the proposed SPA-DF receiver.

Following the schematic of Fig. 1, the user k output of the parallel branch l ($l = 1, \ \ldots, \ L$) for the SPA-DF receiver structure is given by:

$$z_k^l(i) = \mathbf{w}_k^H(i)\mathbf{r}(i) - [\mathbf{M}_l \mathbf{F}]_k^H \hat{\mathbf{b}}(i) \qquad (19)$$

where $\hat{\mathbf{b}}(i) = \mathrm{sgn}[\Re(\mathbf{W}^H \mathbf{r}(i))]$ and the matrices $\mathbf{M}_l$ are permutated square identity ($\mathbf{I}_K$) matrices with dimension K whose structures for an $L = 4$-branch SPA-DF scheme are given by:

$$\mathbf{M}_1 = \mathbf{I}_K, \ \mathbf{M}_2 = \begin{bmatrix} \mathbf{0}_{K/4,3K/4} & \mathbf{I}_{3K/4} \\ \mathbf{I}_{K/4} & \mathbf{0}_{K/4,3K/4} \end{bmatrix},$$

$$\mathbf{M}_3 = \begin{bmatrix} \mathbf{0}_{K/2} & \mathbf{I}_{K/2} \\ \mathbf{I}_{K/2} & \mathbf{0}_{K/2} \end{bmatrix}, \ \mathbf{M}_4 = \begin{bmatrix} 0 & \ldots & 1 \\ \vdots & \cdots & \vdots \\ 1 & \ldots & 0 \end{bmatrix} \qquad (20)$$

where $\mathbf{0}_{m,n}$ denotes an $m \times n$-dimensional matrix full of zeros and the structures of the matrices M_l correspond to phase shifts regarding the cancellation order of the users. The purpose of the matrices in (20) is to change the order of cancellation. When $\mathbf{M} = \mathbf{I}$ the order of cancellation is a simple successive cancellation (S-DF) based upon the user powers (the same as [9], [10]). Specifically, the above matrices perform the cancellation with the following order with respect to user powers: M_1 with indices $1, \ldots, K$; M_2 with indices $K/4, K/4+1, \ldots, K, 1, \ldots, K/4-1$;$M_3$ with indices $K/2, K/2+1, \ldots, K, 1, \ldots, K/2-1$; M_4 with $K, \ldots, 1$ (reverse order). The proposed ordering algorithm shifts the ordering of the users according to K/B, where B is the number of parallel branches. The rationale for this approach is to shift the ordering and attempt to benefit a given user or group of users for each decoding branch. Following this approach, a user that for a given ordering appears to be in an unfavorable position can benefit in other parallel branches by being detected in a more favorable situation. For more branches, additional phase shifts are applied with respect to user cancellation ordering. Note that different update

orders were tested although they did not result in performance improvements.

The final output $\hat{b}_k^f(i)$ of the SPA-DF detector chooses the best estimate of the L candidates for each symbol interval i as described by:

$$\hat{b}_k^{(f)}(i) = \mathrm{sgn}\Big[\Re\Big(\arg\min_{1\leq l\leq L} e_k^l(i)\Big)\Big] \qquad (21)$$

where the best estimate is the value $z_k^l(i)$ that minimizes $e_k^l(i) = |b_k(i) - z_k^l(i)|$ and $\hat{b}_k^{(f)}(i)$ forms the vector of final decisions $\hat{\mathbf{b}}_k^{(f)}(i) = [\hat{b}_1^{(f)}(i) \ \ldots \ \hat{b}_K^{(f)}(i)]^T$. The number of parallel branches L that yield detection candidates is a parameter that must be chosen by the designer. In this context, the optimal ordering algorithm conducts an exhaustive search and is given by

$$\hat{b}_k^{(f)}(i) = \mathrm{sgn}\Big[\Re\Big(\arg\min_{1\leq l\leq K!} e_k^l(i)\Big)\Big] \qquad (22)$$

where the number of candidates is $L = K!$ and is clearly very complex for practical systems. Our studies indicate that $L = 4$ achieves most of the gains of the new structure and offers a good trade-off between performance and complexity. The SPA-DF system employs the same filters, namely $\mathbf{W}$ and $\mathbf{F}$, of the traditional S-DF structure and requires additional arithmetic operations to compute the parallel arbitrated candidates. A discussion of the approximate MMSE attained by the proposed SPA-DF structure is included in Appendix II, whereas expressions for the MMSE of the optimal ordering algorithm are given in Appendix III. As occurs with S-DF receivers, a disadvantage of the SPA-DF detector is that it generally does not provide uniform performance over the user population. In a scenario with tight power control successive techniques tend to favor the last detected users, resulting in non-uniform performance. To equalize the performance of the users an iterative technique with multiple stages can be used.

B. Iterative Successive Parallel Arbitrated DF Detection

In [14], Woodward *et al.* presented an iterative detector with an S-DF in the first stage and P-DF or S-DF structures, with users being demodulated in reverse order, in the second stage. The work of [14] was then extended to account for coded systems and training-based reduced-rank filters [15]. Here, we focus on the proposed SPA-DF receiver and the low complexity near-optimal ordering algorithm, and combine the SPA-DF structure with iterative detection. An iterative receiver with hard-decision feedback is defined by:

$$\mathbf{z}^{(m+1)}(i) = \mathbf{W}^H(i)\mathbf{r}(i) - \mathbf{F}^H(i)\hat{\mathbf{b}}^{(m)}(i) \qquad (23)$$

where the filters $\mathbf{W}$ and $\mathbf{F}$ can be S-DF or P-DF structures, and $\hat{\mathbf{b}}^m(i)$ is the vector of tentative decisions from the preceding iteration that is described by:

$$\hat{\mathbf{b}}^{(1)}(i) = \mathrm{sgn}\Big(\Re\Big[\mathbf{W}^H(i)\mathbf{r}(i)\Big]\Big) \qquad (24)$$

$$\hat{\mathbf{b}}^{(m)}(i) = \mathrm{sgn}\Big(\Re\Big[\mathbf{z}^{(m)}(i)\Big]\Big), \ m > 1 \qquad (25)$$

where the number of stages m depends on the application. More stages can be added and the order of the users is reversed from stage to stage.

To equalize the performance over the user population, we consider a two-stage structure. The first stage is an SPA-DF scheme with filters $\mathbf{W}^1$ and $\mathbf{F}^1$. The tentative decisions are passed to the second stage, which consists of an S-DF, an P-DF or an SPA-DF detector with filters $\mathbf{W}^2$ and $\mathbf{F}^2$, that are computed similarly to $\mathbf{W}^1$ and $\mathbf{F}^1$ but use the decisions of the first stage. The resulting iterative receiver system is denoted ISPAS-DF when an S-DF scheme is deployed in the second stage, whereas for P-DF filters in the second stage the overall scheme is called ISPAP-DF. The output of the second stage of the resulting scheme is:

$$z_j^{(2)}(i) = [\mathbf{M}\mathbf{W}^2(i)]_j^H\mathbf{r}(i) - [\mathbf{M}\mathbf{F}^2(i)]_j^H\hat{\mathbf{b}}^{(2)}(i) \qquad (26)$$

where z_j is the jth component of the soft output vector $\mathbf{z}$, $\mathbf{M}$ is a square permutation matrix with ones along the reverse diagonal and zeros elsewhere (similar to $\mathbf{M}_4$ in (18)), $[.]_j$ denotes the jth column of the argument (a matrix), and $\hat{b}_j^m(i) = \mathrm{sgn}[\Re(z_j^m(i))]$. The third proposed iterative scheme is denoted ISPASPA-DF and corresponds to an SPA-DF architecture employed in both stages. The output of the lth branch of its second stage is:

$$z_{l,j}^{(2)}(i) = [\mathbf{M}\mathbf{W}^2(i)]_j^H\mathbf{r}(i) - [\mathbf{M}_l\mathbf{F}^2(i)]_j^H\hat{\mathbf{b}}^{(2)}(i) \qquad (27)$$

where $\hat{b}_j^{(2)}(i) = \mathrm{sgn}\Big[\Re\Big(\arg\min_{1\leq l\leq L} e_{l,j}^l(i)\Big)\Big]$ and $e_{l,j} = |b_k(i) - z_{l,j}(i)|$. Note that the users in the second stage are demodulated successively and in reverse order relative to the first branch of the SPA-DF structure (a conventional S-DF). The role of reversing the cancellation order in successive stages is to equalize the performance of the users over the population or at least reduce the performance disparities. Indeed, it provides a better performance than keeping the same ordering as the last decoded users in the first stage tend to be favored by the reduced interference. The rationale is that by using these benefited users (last decoded ones) as the first ones to be decoded in the second stage, the resulting performance is improved. Additional stages can be included, although our studies suggest that the gains in performance are marginal. Hence, the two-stage scheme is adopted for the rest of this work.

V. Successive Parallel Arbitrated DF and Iterative Detection for Coded Systems

This section is devoted to the description of the proposed SPA-DF detector and iterative detection schemes for coded systems which employ convolutional codes with Viterbi and turbo decoding. Specifically, we present iterative DF detectors based on the proposed SPA-DF structure which exploits user ordering and combine the SPA-DF with either the S-DF, the P-DF or another SPA-DF in the second stage. We show that a reduced number of turbo iterations can be used with the proposed iterative detector when a near-optimal user ordering is employed and that savings in transmitted power are also obtained as compared to previously reported turbo detectors [19]-[23].

A. Convolutional Codes with Viterbi Decoding

The structure shown in Fig. 1 can be extended to coded systems by including a decoder after the selection unit and before the slicer and an encoder that processes the refined estimates before the feedback filter $\mathbf{F}(i)$. For the proposed SPA-DF receiver structure, users are decoded successively with the aid of the Viterbi algorithm for each parallel arbitrated branch and then reencoded with a convolutional encoder and used for interference cancellation. The motivation for the proposed encoded structure is that significant gains can be obtained from iterative techniques with soft cancellation methods and error control coding [17]-[23] and from efficient receivers structures and ordering algorithms such as the novel SPA-DF detector. The decoding process of the existing S-DF, P-DF and iterative schemes, namely the ISS-DF and the ISP-DF, are explained in [14]. The decoding of the proposed iterative detection schemes that employ the SPA-DF detector (ISPAS-DF, ISPAP-DF and ISPASPA-DF) resembles the uncoded case, where the second stage benefits from the enhanced estimates provided by the first stage that now employs convolutional codes followed by a Viterbi decoder with branch metrics based on the Hamming distance. Specifically, the output of the second stage of the resulting scheme for coded systems is:

$$z_j^{(2)}(i) = [\mathbf{M}\mathbf{W}^2(i)]_j^H \mathbf{r}(i) - [\mathbf{M}\mathbf{F}^2(i)]_j^H \hat{\mathbf{b}}^{(2)}(i) \qquad (28)$$

where

$$[\hat{\mathbf{b}}^{(2)}(i)]_l = \begin{cases} \hat{b}_j^{(2)} & \text{for } l > j \\ \hat{b}_j^{(1)} & \text{for } l < j \end{cases} \qquad (29)$$

where $[\hat{\mathbf{b}}^{(2)}(i)]_l$ is the lth entry of the decision vector $\hat{\mathbf{b}}^{(2)}(i)$. Accordingly, the output of the second stage of the ISPASPA-DF (the SPA-DF architecture is employed in both stages) is desbribed by:

$$z_{l,j}^{(2)}(i) = [\mathbf{M}\mathbf{W}^2(i)]_j^H \mathbf{r}(i) - [\mathbf{M}_l\mathbf{F}^2(i)]_j^H \hat{\mathbf{b}}^{(2)}(i) \qquad (30)$$

where $\hat{b}_j^{(2)}(i) = \operatorname{sgn}\Big[\Re\Big(\arg\min_{1\leq l\leq L} e_{l,j}(i)\Big)\Big]$ and

$$e_{l,j}(i) = \begin{cases} |b_j^{(2)}(i) - z_{l,j}(i)| & \text{for } l > j \\ |b_j^{(1)}(i) - z_{l,j}(i)| & \text{for } l < j \end{cases} \qquad (31)$$

B. Iterative Turbo Receiver and Decoding

A CDMA system with convolutional codes being used at the transmitter and the proposed iterative SPA-DF receiver with turbo decoding is illustrated in Fig. 2. The proposed iterative (turbo) receiver structure consists of the following stages: a soft-input-soft-output (SISO) SPA-DF detector and a maximum *a posteriori* (MAP) decoder. These stages are separated by interleavers and deinterleavers. Specifically, soft outputs from the SPA-DF are used to estimate likelihoods which are interleaved and input to the MAP decoder for the convolutional code. The MAP decoder computes *a posteriori* probabilities (APPs) for each user's encoded symbols, which are used to generate soft estimates. These soft estimates are subsequently used to update the SPA-DF filters, de-interleaved and fed back through the feedback filter. This process is then iterated.

The proposed SPA-DF detector yields the *a posteriori* log-likelihood ratio (LLR) of a transmitted symbol ($+1$ or -1) for every code bit of each user as given by

$$\Lambda_1[b_k(i)] = \log\frac{P[b_k(i)=+1|\mathbf{r}(i)]}{P[b_k(i)=-1|\mathbf{r}(i)]}, \quad k = 1,\ldots,K. \qquad (32)$$

Using Bayes' rule, the above equation can be written as

$$\begin{aligned}\Lambda_1[b_k(i)] &= \log\frac{P[\mathbf{r}(i)|b_k(i)=+1]}{P[\mathbf{r}(i)|b_k(i)=-1]} + \log\frac{P[b_k(i)=+1]}{P[b_k(i)=-1]} \\ &= \lambda_1[b_k(i)] + \lambda_2^p[b_k(i)]\end{aligned} \qquad (33)$$

where $\lambda_2^p[b_k(i)] = \log\frac{P[b_k(i)=+1]}{P[b_k(i)=-1]}$ represents the *a priori* LLR of the code bit $b_k(i)$, which is computed by the MAP decoder of the kth user in the previous iteration, interleaved and then fed back to the SPA-DF detector. Note that the superscript ^{p} denotes the quantity obtained in the previous iteration. Assuming equally likely bits, for the first iteration we have $\lambda_2^p[b_k(i)] = 0$ for all users. The first term in (33), i.e. $\lambda_1[b_k(i)] = \log\frac{P[\mathbf{r}(i)|b_k(i)=+1]}{P[\mathbf{r}(i)|b_k(i)=-1]}$, represents the *extrinsic* information yielded by the SISO SPA-DF detector based on the received data $\mathbf{r}(i)$, the prior information about the code bits of all other users $\lambda_2^p[b_l(i)], l \neq k$ and the prior information about the code bits of the kth user other than the ith bit. The extrinsic information $\lambda_1[b_k(i)]$ provided by the MAP decoder is then de-interleaved and fed back into the MAP decoder of the kth user as the *a priori* information in the next iteration.

Based on the prior information $\lambda_1^p[b_k(i)]$ and the trellis structure of the code, the kth user's MAP decoder computes the *a posteriori* LLR of each code bit as described by

$$\begin{aligned}\Lambda_2[b_k(i)] &= \log\frac{P[b_k(i)=+1|\lambda_1^p[b_k(i);\text{decoding}]}{P[b_k(i)=-1|\lambda_1^p[b_k(i);\text{decoding}]} \\ &= \lambda_2[b_k(i)] + \lambda_1^p[b_k(i)], \quad k=1,\ldots,K.\end{aligned} \qquad (34)$$

From the above equality, it is seen that the output of the MAP decoder is the sum of the prior information $\lambda_1^p[b_k(i)]$ and the extrinsic information $\lambda_2[b_k(i)]$ yielded by the MAP decoder. This extrinsic information is the information about the code bit $b_k(i)$ obtained from the prior information about the other code bits $\lambda_1^p[b_k(j)],\ j \neq i$ [22]. The MAP decoder also computes the *a posteriori* LLR of every information bit, which is used to make a decision on the decoded bit at the last iteration. After interleaving, the extrinsic information yielded by the K MAP decoders $\lambda_2[b_k(i)],\ k = 1,\ldots,K$ is fed back to the SPA-DF detector, as the prior information about the code bits of all users in the subsequent iteration. At the first iteration, the extrinsic information $\lambda_1[b_k(i)]$ and $\lambda_2[b_k(i)]$ are statistically independent and as the iterations are computed they become more correlated and the improvement due to each iteration is gradually reduced.

For the purpose of MAP decoding, we assume that the interference plus noise at the output of the subtractor in Fig. 2 (b), which corresponds to $\mathbf{z}(i)$, is Gaussian. This assumption is reasonable when there are many active users, has been used in previous works [15],[22]-[23] and provides an efficient and accurate way of computing the extrinsic information. Thus, for

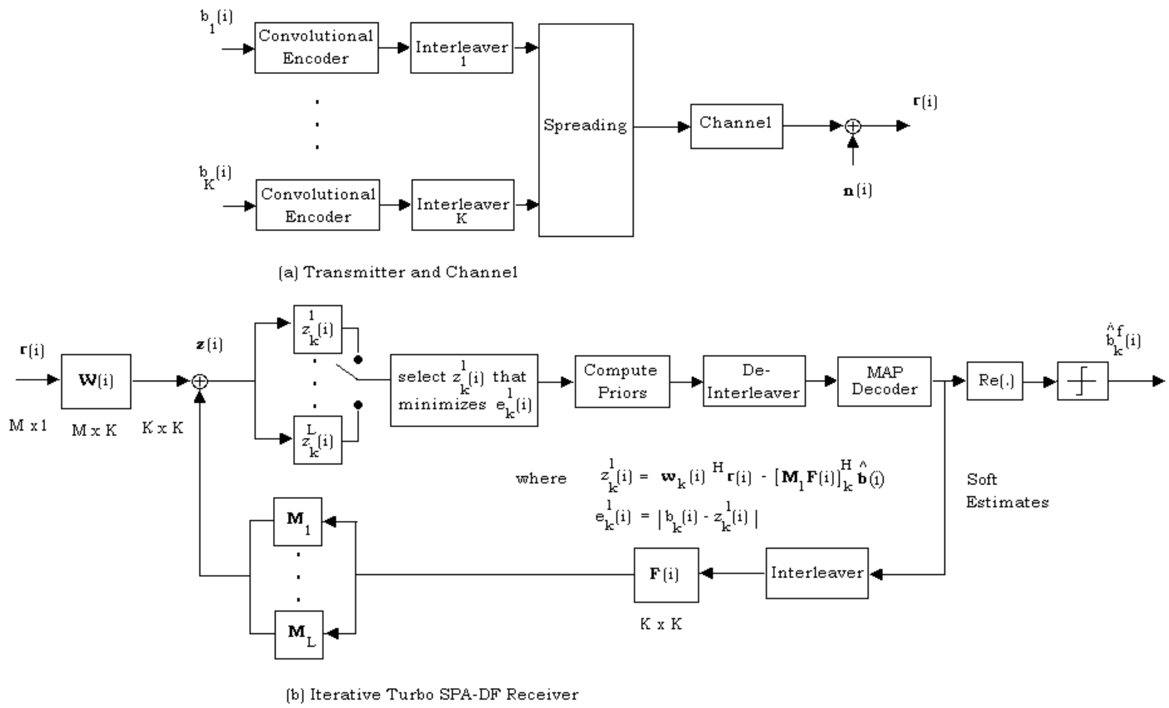

Fig. 2. Block diagram of the proposed system with the SPA-DF detector and turbo decoding.

the kth user and mth iteration the soft output of the SPA-DF detector is written as

$$z_k^{(m)}(i) = V_k^{(m)} b_k(i) + \xi_k^{(m)}(i) \qquad (35)$$

where $V_k^{(m)}(i)$ is a scalar variable equivalent to the kth user's amplitude and $\xi_k^{(m)}(i)$ is a Gaussian random variable with variance $\sigma^2_{\xi_k^{(m)}}$. Since we have

$$V_k^{(m)}(i) = E\big[b_k^*(i) z_k^{(m)}(i)\big] \qquad (36)$$

and

$$\sigma^2_{\xi_k^{(m)}}(i) = E\big[|z_k^{(m)}(i) - V_k^{(m)}(i) b_k(i)|^2\big] \qquad (37)$$

the designer can obtain the estimates $\hat{V}_k^{(m)}(i)$ and $\hat{\sigma}^2_{\xi_k^{(m)}}(i)$ via the corresponding sample averages over the packet transmission. These estimates are used to compute the detector *a posteriori* probabilities $P[b_k(i) = \pm 1 | z_k^{(m)}(i)]$ which are de-interleaved and input to the MAP decoder for the convolutional code. In what follows, we assume that the MAP decoder generates APPs $P[b_k(i) = \pm 1]$, which are used to compute the input to the feedback filter $\mathbf{f}_k(i)$. From (35) the extrinsic information delivered by the soft output SPA-DF is given by

$$\lambda_1[b_k(i)] = \log \frac{P[z_k^{(m)}(i)|b_k(i) = +1]}{P[z_k^{(m)}(i)|b_k(i) = -1]} = -\frac{(z_k^{(m)}(i) - V_k^{(m)})^2}{2\sigma^2_{\xi_k^{(m)}}(i)} + \frac{(z_k^{(m)}(i) + V_k^{(m)})^2}{2\sigma^2_{\xi_k^{(m)}}(i)} = \frac{2V_k^{(m)} z_k^{(m)}(i)}{\sigma^2_{\xi_k^{(m)}}(i)} \qquad (38)$$

The SPA-DF turbo detector chooses the best estimate of the L candidates for the mth turbo decoding iteration as:

$$l_{best,k}^{(m)}(i) = \arg \min_{1 \leq l \leq L} e_k^l(i) \qquad (39)$$

where the best estimate is the value $z_k^l(i)$ which minimizes $e_k^l(i) = |b_k(i) - z_k^l(i)|$.

C. Extensions

Here, we briefly comment on how the proposed receiver structures can be extended to take into account asynchronous systems, dynamic scenarios, other types of communications systems and multiple access techniques.

For asynchronous systems with large relative delays amongst the users, the observation window of each user should be expanded in order to consider an increased number of samples derived from the offsets amongst users. Alternatively for small relative delays amongst users, the designer can resort

to chip oversampling to compensate for the random timing offsets. These remedies imply in augmented filter lengths and consequently increased computational complexity. To alleviate for the increase in filter length and the increased amount of training, the designer can resort to reduced-rank estimation techniques such as the Multistage Wiener Filter, as in [14], or to a new very promising technique that employs interpolated FIR filters [25].

An extension with low complexity turbo schemes such as the one in [26] are also possible with the structures presented in this paper. For dynamic channels that are subject to fading, the designer can rely on adaptive signal processing techniques and make the proposed detector structures adaptive in order to track the variations of the channel and the interference. This includes some modifications for CDMA systems with long codes, which require a different approach for estimating the covariance observation matrix $\mathbf{R}$ due to the loss of the cyclostationarity.

Finally, we also remark that the proposed detection schemes can be deployed for narrow-band systems with multiple transmitter and receiver antennas, exploiting the capacity improvements of spatial multiplexing.

VI. SIMULATIONS

In this section, we evaluate the performance of the iterative arbitrated DF structures introduced in Section IV and compare them with other existing structures. Due to the extreme difficulty of theoretically analyzing such scheme, we adopt a simulation approach and conduct several experiments in order to verify the effectiveness of the proposed techniques. In particular, we have carried out experiments to assess the bit error rate (BER) performance of the DF receivers for different loads, channel profiles, and signal to noise ratios (E_b/N_0). The DS-CDMA system employs random generated spreading sequences of length $N = 16$, $N = 32$ and $N = 64$, has perfect power control and use statistically independent random channels with $L_p = 3$, whose coefficients $h_{k,l}$ are taken, for each run, from uniform random variables between -1 and 1, and which are normalized so that $\sum_{l=1}^{L_p} h_{k,l}^2 = 1$. It should be remarked that the existence of multipath creates an error floor for the multiuser receivers, making it more difficult the interference suppression of associated users. Note also that given the performance of current power control algorithms, ideal power control is not far from a realistic situation. The matrices used in (14) and (15) are estimated by $\hat{\mathbf{R}}(i) = \frac{1}{i}\sum_{l=1}^{i} \mathbf{r}(l)\mathbf{r}^H(l)$ and $\hat{\mathbf{B}}(i) = \frac{1}{i}\sum_{l=1}^{i} \mathbf{r}(l)\hat{\mathbf{b}}^H(l)$. For coded systems, we employ a convolutional code with rate $R = 3/4$ and constraint length 6 which can be found in [24]. In particular, for turbo decoding plots we used S-random interleavers with block size equal to 256. In the following experiments, averaged over 200 runs for uncoded systems, over 2000 for encoded systems with Viterbi decoding and over 20000 for turbo decoded schemes, it is indicated the receiver structure (linear or decision feedback (DF)). Amongst the different DF structures, we consider:

- S-DF: the successive DF detector of [9], [10].
- P-DF: the parallel DF detector of [13], [14].
- ISS-DF: the iterative system of Woodward *et al.* [14] with S-DF in the first and second stages.
- ISP-DF: the iterative system of Woodward *et al.* [14] with S-DF in the first stage and P-DF in the second stage.
- SPA-DF: the proposed successive parallel arbitrated receiver.
- ISPAS-DF: the proposed iterative detector with the novel SPA-DF in the first stage and the S-DF in the second stage.
- ISPAP-DF: the proposed iterative receiver with the SPA-DF in the first stage and the P-DF in the second stage.
- ISPASPA-DF: the proposed iterative receiver with the SPA-DF in the first and second stages.

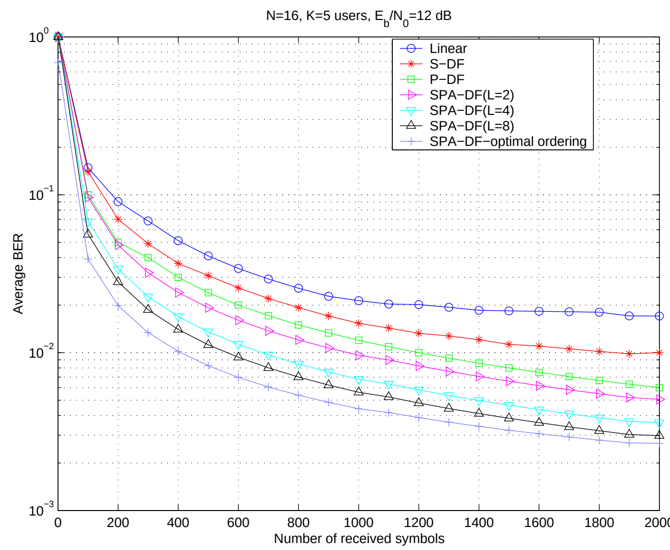

Fig. 3. BER performance versus number of symbols.

Let us first consider the proposed SPA-DF, evaluate the number of arbitrated branches that should be used in the ordering algorithm and account for the impact of additional branches upon performance. In addition to this, we carry out a comparison of the proposed low complexity user ordering algorithm against the optimal ordering approach, briefly described in Section IV. A, that tests $K!$ possible branches and selects the most likely estimate. We designed the novel DF receivers with $L = 2, 4, 8$ parallel branches and compared their BER performance versus number of symbols with the existing S-DF and P-DF structures, as depicted in Fig. 3. The results show that the proposed low complexity ordering algorithm achieves a performance close to the optimal ordering, whilst keeping the complexity reasonably low for practical utilization. Furthermore, the performance of the new SPA-DF scheme with $L = 2, 4, 8$ outperforms the S-DF and the P-DF detector. It can be noted from the curves that the performance of the new SPA-DF improves as the number of parallel branches increase. In this regard, we also notice that the gains of performance obtained through additional branches decrease as L is increased, resulting in marginal improvements for more than $L = 4$ branches. For this reason, we adopt $L = 4$ for the remaining experiments because it presents a very attractive trade-off between performance and complexity.

A performance comparison in terms of BER of the proposed DF structures, namely SPA-DF, ISPAP-DF, ISPAS-DF and ISPASPA-DF with existing iterative and conventional DF and linear detectors is illustrated in Figs. 4 to 5, for uncoded systems and in Fig. 6, for convolutionally coded systems. In particular, we show BER performance curves versus E_b/N_0 and number of users (K) for the analyzed receivers. The results for a system with $N = 32$, depicted in Fig. 4 indicate that the best performance is achieved with the novel ISPASPA-DF (the SPA-DF is employed in two cascaded stages), followed by the new ISPAP-DF, the existing ISP-DF [14], the ISPAS-DF, the SPA-DF, the P-DF, the ISS-DF, the S-DF and the linear detector. Specifically, the ISPASPA-DF detector can save up to 1.5 dB and support up to 4 more users in comparison with the ISP-DF (which is the best existing scheme) for the same BER performance. The ISPAP-DF scheme can save up to 1 dB and support up to 2 more users in comparison with the ISP-DF for the same BER performance. Moreover, the performance advantages of the ISPASPA-DF and ISPAP-DF systems are substantially superior to the other existing approaches.

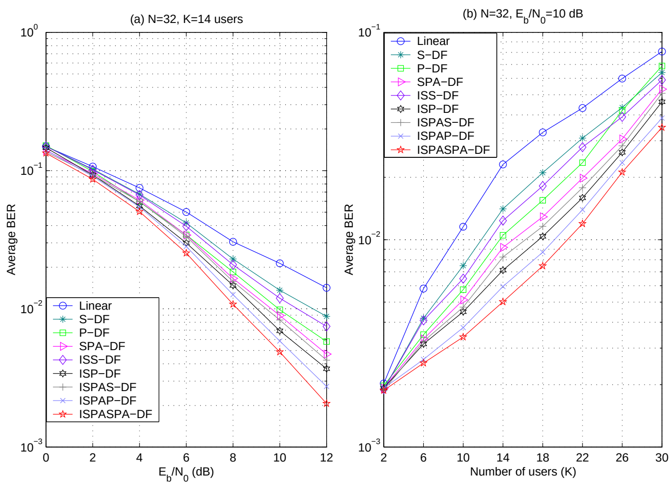

Fig. 4. BER performance versus (a) E_b/N_0 and (b) number of users (K).

The results for a larger system with $N = 64$, illustrated in Fig. 5, corroborate the curves obtained for the smaller system in Fig. 4. In particular, the same BER performance hierarchy is observed for the detection schemes (except for the ISPAS-DF, that now outperforms the ISP-DF) and we notice some additional gains in performance for the proposed schemes over the existing techniques. Specifically, the ISPASPA-DF detector can save up to 1.8 dB and support up to 10 additional users in comparison with the ISP-DF for the same BER performance. The ISPAP-DF scheme can save up to 1.4 dB and support up to 8 more users in comparison with the ISP-DF for the same BER performance. Moreover, the performance advantages of the ISPASPA-DF and ISPAP-DF systems are even more pronounced over the other analyzed schemes for larger systems.

The BER performance of the analyzed detection schemes was then examined for convolutionally encoded systems with Viterbi decoding, $N = 32$ and rate $R = 3/4$, as depicted in Fig. 6. The results corroborate those obtained for uncoded

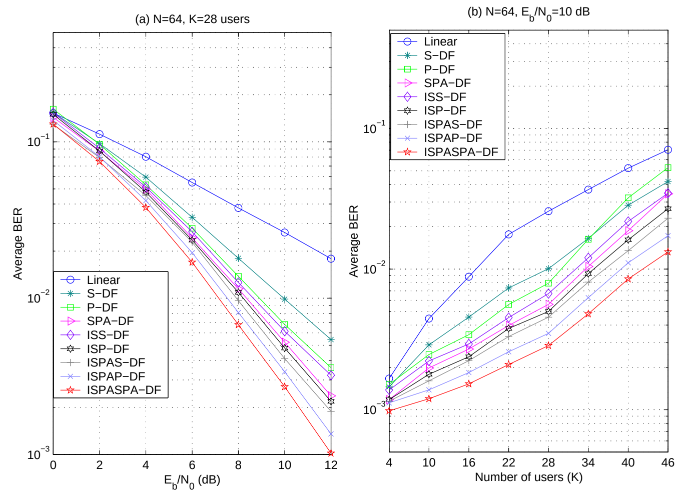

Fig. 5. BER performance versus (a) E_b/N_0 and (b) number of users (K).

systems in Figs. 4 and 5, and indicate that the proposed ISPASPA-DF and ISPAP-DF detection schemes significantly outperform the remaining receiver structures. In particular, the ISPASPA-DF detector can support up to 8 additional users in comparison with the ISP-DF for the same BER performance, whereas the ISPAP-DF scheme can accomodate up to 6 more users in comparison with the ISP-DF for the same BER performance. It is worth noting that the linear and P-DF detectors experience performance losses for coded systems, relative to the other structures, as verified in [14] and which is a result of the loss in spreading gain that increases the interference power at the output of the MMSE receiver.

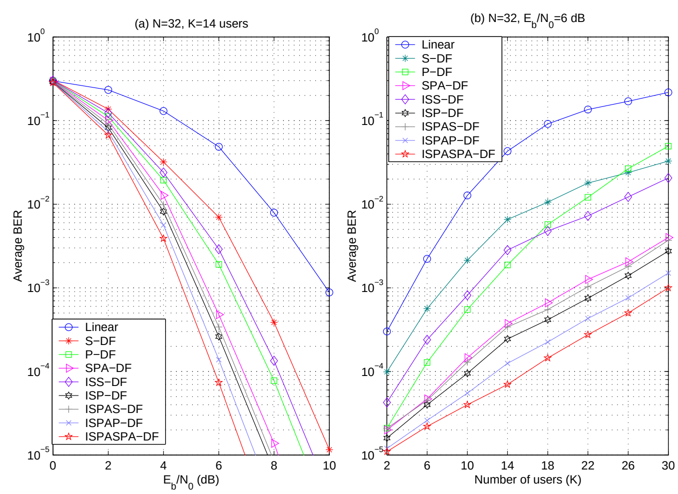

Fig. 6. BER performance of a convolutionally coded system with $R = 3/4$ versus (a) E_b/N_0 and (b) number of users (K).

The BER performance of the analyzed detection schemes was also investigated for convolutionally encoded systems with turbo decoding. In our studies with turbo receivers, we tested several code rates and found that $R = 1/2$ was unable to attain good performance for highly loaded systems, whereas $R = 3/4$ was powerful enough to obtain good performance

even in fully loaded systems. For this reason, we adopted the rate $R = 3/4$ for the remaining experiments with iterative decoders and considered a system with $N = 32$, as depicted in Fig. 7. The results corroborate those obtained for uncoded and encoded systems with Viterbi decoding in Figs. 5 and 6, and indicate that the proposed ISPASPA-DF and ISPAP-DF detection schemes significantly outperform the remaining receiver structures. In particular, the ISPASPA-DF detector can approach the single user bound with only 4 iterations and offer a significant advantage over the existing detectors. In comparison with existing iterative DF detectors, the ISPASPA-DF can save up to 0.5 dB for the same BER performance, whereas it can accommodate a fully loaded system with only 4 iterations and operating with only 4 dB with negligible performance degradation as the load is increased.

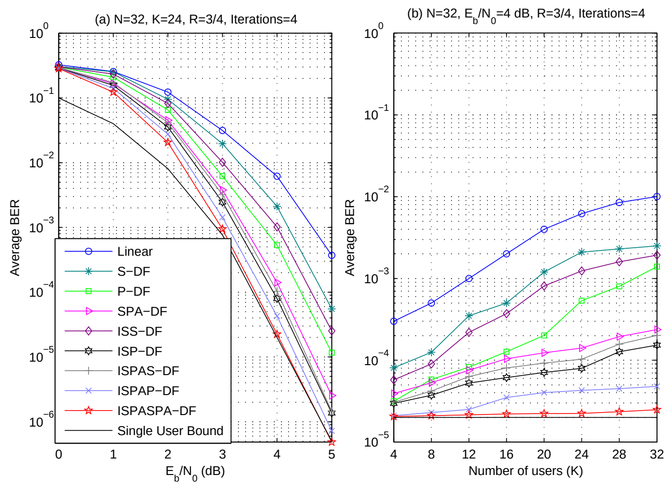

Fig. 7. BER performance of a turbo decoded system with $R = 3/4$ versus (a) E_b/N_0 and (b) number of users (K).

In Fig. 8 it is illustrated the average BER performance of the detectors versus the number of iterations of the turbo decoder. The plots show that the proposed ISPASPA-DF and the ISPAP-DF detectors achieve the single user bound with only 4 and 7 iterations, respectively, whereas the remaining detectors require more iterations to achieve this performance. This is an important feature of the proposed detectors as they can save considerable computational resources by operating with a lower number of turbo iterations.

The last scenario, shown in Figs. 9, considers the individual BER performance of the users for both uncoded and convolutionally encoded systems with Viterbi decoding. From the curves, we observe that a disadvantage of S-DF relative to P-DF is that it does not provide uniform performance over the user population. We also notice that for the S-DF receivers, user 1 achieves the same performance of their linear receivers counterparts, and as the successive cancellation is performed users with higher indices benefit from the interference cancellation. The same non-uniform performance is verified for the proposed SPA-DF, the existing ISS-DF and the novel ISPAS-DF and ISPASPA-DF. Conversely, the new ISPAP-DF, the existing P-DF and the existing ISP-DF provide uniform

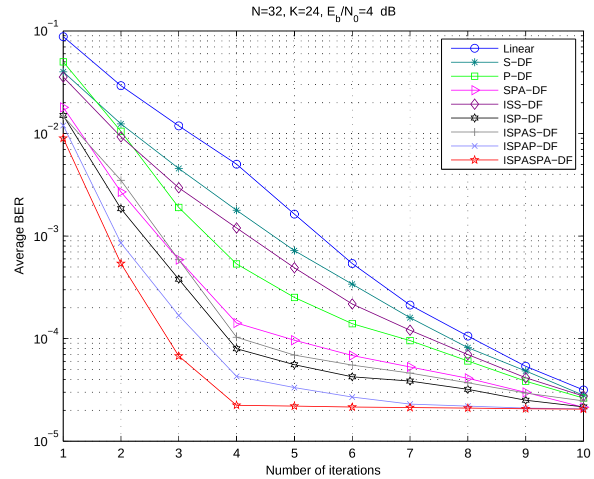

Fig. 8. BER performance of a turbo decoded system with $R = 3/4$ versus number of iterations.

performance over the users which is an important goal for the uplink of DS-CDMA systems. In particular, the novel ISPAP-DF detector achieves the best uniform performance of the analyzed structures and is superior to the ISP-DF and to the P-DF, that suffers from error propagation. For coded systems, we notice that the performance of the proposed ISPASPA-DF and ISPAS-DF, and the existing ISS-DF and S-DF becomes very attractive for the users with indices greater than 5 (where the SIC-based schemes outperform the ISPAP-DF, the ISP-DF and the P-DF). This suggests the deployment of these structures for systems that rely on differentiated services, where the quality of service (QoS) can be made different for different groups of users. In this context and as an example, users with the first indices and poorer performance should be allocated to voice services, while the users with better performance should be designated to data transmission services that require improved QoS.

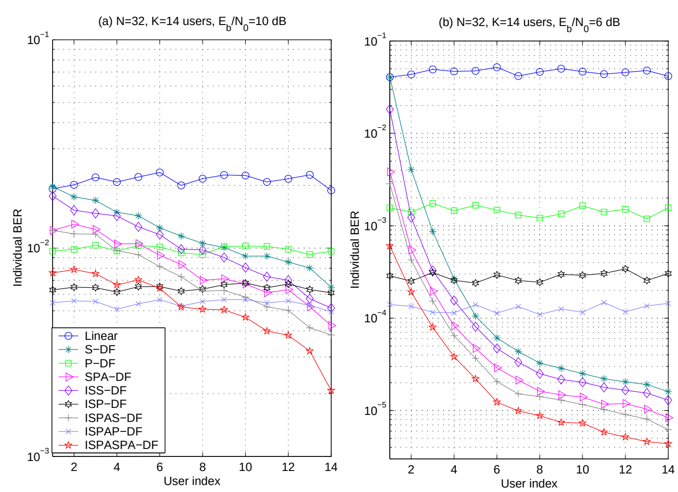

Fig. 9. BER performance versus user index for (a) an uncoded system (b) a convolutionally coded system with rate $R = 3/4$.

VII. CONCLUSIONS

A novel SPA-DF structure and a low complexity near-optimal ordering algorithm were presented and combined with iterative techniques for use with cascaded DF stages for mitigating the deleterious effects of error propagation. The proposed SPA-DF and iterative receivers for DS-CDMA systems were investigated in an uplink scenario and compared to existing schemes in the literature. The results for both uncoded and convolutionally encoded systems using Viterbi and turbo decoding show that the new detection schemes can offer considerable gains as compared to existing DF and linear receivers, support systems with higher loads and mitigate the phenomenon of error propagation.

APPENDIX

In this Appendix, we provide some relationships between the MMSE attained by a decision feedback structure with perfect and imperfect feedback. Let us consider an alternative expression for the cost function in (4) for user k:

$$\begin{aligned} J_{MSE} = \sigma_b^2 - \mathbf{w}_k^H\mathbf{p}_k - \mathbf{p}_k^H\mathbf{w}_k + \mathbf{w}_k^H\mathbf{R}\mathbf{w}_k + \mathbf{f}_k^H\mathbf{f}_k - \mathbf{w}_k^H\mathbf{B}\mathbf{f}_k \\ - \mathbf{f}^H\mathbf{B}^H\mathbf{w}_k \end{aligned} \tag{40}$$

Consider the expression for the feedforward filter $\mathbf{w}_k = \mathbf{R}^{-1}(\mathbf{p}_k + \mathbf{B}\mathbf{f}_k)$ obtained in (16) and the expression for the feedback filter $\mathbf{f}_k = \mathbf{Q}^{-1}\mathbf{B}^H\mathbf{w}_k$ with $\mathbf{Q} = E[\hat{\mathbf{b}}\hat{\mathbf{b}}^H]$ in (17). By substituting the optimal MMSE expressions obtained in (17) into (16) for the filters we obtain an alternative expression for the feedback filter $\mathbf{f}_k$:

$$\mathbf{f}_k = \mathbf{D}^{-1}\mathbf{Q}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \tag{41}$$

where $\mathbf{D} = (\mathbf{I} - \mathbf{Q}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{B})$ and the above expression only depends on $\mathbf{Q}$, $\mathbf{B}$, $\mathbf{R}$ and $\mathbf{p}_k$. By inserting the expression $\mathbf{w}_k = \mathbf{R}^{-1}(\mathbf{p}_k + \mathbf{B}\mathbf{f}_k)$ and (41) into (40), we have for user k:

$$\begin{aligned} J_{MMSE} &= \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{p}_k - \mathbf{f}_k^H\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{f}_k \\ &\quad - \mathbf{f}_k^H\mathbf{B}^H\mathbf{R}^{-1}\mathbf{B}\mathbf{f}_k + \mathbf{f}_k^H\mathbf{f}_k \\ &= \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{p}_k - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{Q}^{-1}\mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\quad - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{Q}^{-1}\mathbf{D}^{-1}\mathbf{Q}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\quad - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{Q}^{-1}\mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{B}\mathbf{D}^{-1}\mathbf{Q}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\quad + \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{Q}^{-1}\mathbf{D}^{-1}\mathbf{Q}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &= \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{p}_k - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{Q}^{-1}\mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\quad - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{Q}^{-1}\mathbf{D}^{-1}\mathbf{Q}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\quad + \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{Q}^{-1}\mathbf{D}^{-1}(\mathbf{I} - \mathbf{B}^H\mathbf{R}^{-1}\mathbf{B})\mathbf{D}^{-1}\mathbf{Q}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \end{aligned} \tag{42}$$

At this point, it is convenient to adopt the judicious approximation $\mathbf{Q} = E[\hat{\mathbf{b}}\hat{\mathbf{b}}^H] \approx \mathbf{I}$, which is justified for moderate to low BER values. By using this approximation we have $\mathbf{f}_k \approx \mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k$, where $\mathbf{D} \approx (\mathbf{I} - \mathbf{B}^H\mathbf{R}^{-1}\mathbf{B})$, and the MMSE expression for user k is approximated by:

$$\begin{aligned} J_{MMSE} &\approx \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{p}_k - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\quad - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\quad + \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{D}^{-1}(\mathbf{I} - \mathbf{B}^H\mathbf{R}^{-1}\mathbf{B})\mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\approx \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{p}_k - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\quad - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\quad + \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\approx \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{p}_k - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\approx \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\quad - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}(\mathbf{I} - \mathbf{B}^H\mathbf{R}^{-1}\mathbf{B})^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \end{aligned} \tag{43}$$

The approximate expression obtained in (43) represents the MMSE attained by a general decision feedback structure that has imperfect feedback. The equation in (43) is a function of $\mathbf{B}$, $\mathbf{R}$ and $\mathbf{p}_k$, and is still dependent on the decisions. Let us now assume perfect feedback ($\mathbf{b} = \hat{\mathbf{b}}$) and look at the filter expressions. Since $\mathbf{w}_k = \mathbf{R}^{-1}(\mathbf{p}_k + \mathbf{B}\mathbf{f}_k)$ and $\mathbf{f}_k = \mathbf{B}^H\mathbf{w}_k = \mathbf{p}^H\mathbf{R}^{-1}(\mathbf{p}_k + \mathbf{B}\mathbf{f}_k)$

$$\begin{aligned} J_{MMSE} &\approx \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{p}_k - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{D}^{-1}\mathbf{B}^H\mathbf{R}^{-1}\mathbf{p}_k \\ &\approx \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{p}_k - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{B}\mathbf{f}_k \\ &\approx \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}^{-1}(\mathbf{p}_k + \mathbf{B}\mathbf{f}_k) \\ &\approx \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}^{-1}\mathbf{R}\mathbf{w}_k = \sigma_b^2 - \mathbf{p}_k^H\mathbf{w}_k \end{aligned} \tag{44}$$

The approximate expression obtained in (43) has been significantly simplified due to the assumption of perfect feedback and indicates that the MMSE for user k is a function of $\mathbf{w}_k$. If we consider a decision feedback structure such as successive cancellation (S-DF), use the expression for the feedforward filter $\mathbf{w}_k = \mathbf{R}_U^{-1}\mathbf{p}_k$, the MMSE for user k is approximately given by:

$$J_{MMSE} \approx \sigma_b^2 - \mathbf{p}_k^H\mathbf{R}_U^{-1}\mathbf{p}_k \tag{45}$$

where the above result means that the MMSE attained by user k is proportional to the number of undetected users expressed by the covariance matrix $\mathbf{R}_U$. If we consider a decision feedback structure such as parallel cancellation (P-DF), use the expression for the feedforward filter $\mathbf{w}_k = \mathbf{R}_U^{-1}\mathbf{p}_k = \frac{\mathbf{p}_k}{|A_k|^2+\sigma^2}$, the MMSE for user k is approximately given by:

$$J_{MMSE} \approx \sigma_b^2 - \mathbf{p}_k^H(\mathbf{p}_k\mathbf{p}_k^H + \sigma^2\mathbf{I})^{-1}\mathbf{p}_k \tag{46}$$

Note that the above result corresponds to the single-user bound because we assume that all users (with perfect decision) had been fed back, as in P-DF.

For imperfect feedback, the P-DF is known to be susceptible to error propagation, while the S-DF is more effective in combating these deleterious effects. The proposed SPA-DF employs several versions of S-DF in parallel and chooses the best estimate amongst these parallel branches, resulting in improved performance over the S-DF, as verified in our studies. Here, we mathematically discuss the MMSE of the SPA-DF, under the assumption of perfect feedback. If we consider the SPA-DF with L branches, we have L different groups of undetected users, namely, $U_1,\ U_2,\ \ldots,\ U_L$ and the

associated expression for the feedforward filter $\mathbf{w}_k = \mathbf{R}_{U_l}^{-1}\mathbf{p}_k$, where $l = 1, 2, \ldots, L$. Therefore, the MMSE for user k is approximately given by:

$$J_{MMSE} \approx \arg \min_{1 \leq l \leq L} (MSE_{U_l}) \tag{47}$$

where $MSE_{U_l} = \sigma_b^2 - \mathbf{p}_k^H \mathbf{R}_{U_l}^{-1}\mathbf{p}_k$ and the above expression means that the MMSE attained by user k with the SPA-DF is at least equal to a standard S-DF (with $L = 1$ and approximate MMSE given by (45)). The approximate MMSE in (47) is also proportional to the number of undetected users expressed by the covariance matrix $\mathbf{R}_{U_l}$, but can benefit from different groups of undetected users, by selecting the undetected group of users that yield smaller MSE, resulting in better performance. Indeed, the MMSE of the proposed SPA-DF structure in (47) is upperbounded by the MMSE of the standard S-DF detector given through (45).

Here, we mathematically discuss the MMSE of S-DF detectors with the optimal ordering algorithm. If we consider an exhaustive search over all the possible orderings for an S-DF, we have $K!$ different groups of undetected users or equivalently $K!$ possible orderings. The optimal ordering S-DF can be seen as a generalisation of the proposed SPA-DF structure in which the number of branches is equal to $K!$. Mathematically, for the case of imperfect decisions we have for the optimal ordering S-DF the following expression

$$J_{MMSE} \approx \arg \min_{1 \leq l \leq K!} (J_{MSE,l}) \tag{48}$$

where

$$\begin{aligned} J_{MSE,l} = \sigma_b^2 - \mathbf{p}_{k,l}^H \mathbf{R}^{-1}\mathbf{p}_k - \mathbf{f}_{k,l}^H \mathbf{B}^H \mathbf{R}^{-1}\mathbf{p}_k - \mathbf{p}_k^H \mathbf{R}^{-1}\mathbf{B}\mathbf{f}_{k,l} \\ - \mathbf{f}_{k,l}^H \mathbf{B}^H \mathbf{R}^{-1}\mathbf{B}\mathbf{f}_{k,l} + \mathbf{f}_{k,l}^H \mathbf{f}_{k,l} \end{aligned} \tag{49}$$

The expression in (49) is similar in form to the first line of (42) but depends on the ordering l and the associated feedback filter $\mathbf{f}_{k,l}$. In the case of perfect feedback, the corresponding expression for the feedforward filter is $\mathbf{w}_k = \mathbf{R}_{U_l}^{-1}\mathbf{p}_k$, where $l = 1, 2, \ldots, K!$ and we have $K!$ different groups of undetected users, namely, $U_1, U_2, \ldots, U_{K!}$. Therefore, the MMSE for user k is approximately given by

$$J_{MMSE} \approx \arg \min_{1 \leq l \leq K!} (MSE_{U_l}) \tag{50}$$

where $MSE_{U_l} = \sigma_b^2 - \mathbf{p}_k^H \mathbf{R}_{U_l}^{-1}\mathbf{p}_k$ and the above expression means that the MMSE attained by user k with the optimal ordering is at least equal to a standard S-DF (with $L = 1$ and approximate MMSE given by (45)). The approximate MMSE in (50) is indeed proportional to the number of undetected users expressed by the covariance matrix $\mathbf{R}_{U_l}$. The key point is that the designer searches for all possible groups of undetected users and selects the one which yields the smallest MSE, resulting in better performance. The main problem is that as K increases the complexity becomes prohibitive and its implementation impractical.

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